ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-III

Paper Code: C5T

[Real Function & Introduction to Matric Space]

Answer all the questions

1. Show that the function f defined by $f(x) = \frac{1}{x}$, $x \in [1,\infty)$ is uniformly continuous on $[1,\infty)$.
2. If a function $f:[a,b] \to \mathbb{R}$ is continuous on $[a,b]$ then show that f is bounded on $[a,b]$
3. Prove that $\lim_{x\to 3}([x] - \left[\frac{x}{3}\right]) = 0$.
4. Examine with reason whether $\lim_{x\to 0}$ $(\sin\frac{1}{x} + x\sin\frac{1}{x})$ exit or not.
5. Give examples of a function which is continuous and bounded on $\mathbb{R}$, attains suprimum but not infimum.
6. Give examples of a function which is continuous and bounded on $\mathbb{R}$, attains infimum but not suprimum.
7. Give examples of a function which is continuous and bounded on interval but attains neither its suprimum nor infimum.

8. Let [a,b] be a closed and bounded interval and $f:[a,b] \to \mathbb{R}$ be continuous on [a, b] . If f(a) and f(b) are opposite sign then show that there exits at least a point c in the open interval (a,b) such that f(c) =0
9. Show that the Dirichlet's function is everywhere discontinuous on R .
10. Let [a,b] be a closed and bounded interval and a function R be continuous on [a , b]. If *f*(a) ≠*f* (b) then f attains every value between *f*(a) and *f* (b) at least once *in (a, b).* Is the converse true ? Justify .
11. Give and example of a function defined on an interval *I* such that

(i) *f* has jump discontinuity at a point of I .

(ii) *f* has removable discontinuity at a point of I .

(iii) *f* has infinite discontinuity at a point of I .

12. (i) Prove that for no real value of *k* , the equations $x^3 - 12\,x + k = 0$ has two real roots in [-1,1] .
13. Prove that there does not exist a function φ such that $\varphi(x) = f(x)$ on [0,2] where *f*(x) = x- [x]
14. (e) Prove that $x - \frac{x^3}{x} < \sin x < -\frac{x^3}{6} + \frac{x^5}{120}$ for all x >0 .
15. prove that there exists $x \in \ (\ 0 ,\frac{\pi}{2} \ )$ such that $x = \ cos\ x$
16. (b) $Let\ D \subset \mathbb{R}$ and a function $f\!: D \ \to \mathbb{R}$ be $uniformaly$ continuous onD .if { x_n} be a Cauchy sequence in D then show that { f(x_n)} is a $cauchy$ sequence in $\mathbb{R}$. If we drop the condition ‘'$uniformaty$'', then is the above result hold ? Justify .
17. If f(x) be differentiable at x=a show that

$$\lim_{x\to a} \frac{(x+a)f(x)-2af(a)}{x-a} = f(a) + 2af'(a) .$$

18. State and prove L$agrange's$ $mean\ value\ theor$em .give its geometrical $signifiation$
19. $f\!:$ [0,1] $\to \mathbb{R}$ is continuous on [0,1] and f assumes only rational values . I$f\ f\ \left(\frac{1}{2}\right) = \frac{1}{2}$

20. $f : [0,1] \to \mathbb{R}$ is continuous on [0,1] and f assumes only rational values . If $f\left(\frac{1}{2}\right) = \frac{1}{2}$,

Prove that $f(x) = \frac{1}{2}$ for all x $\in$ [0,1] .

21. Give an examples show that a function which is continuous on an open bounded interval may not be uniformly continuous there .

22. Let f be continuous on closed interval [a, b] and $f(x)$ =0 when x is rational .Show that $f(x)$ =0 for every $x \in$ [a, b]

23. Find f '(0) [if exist] for the function $f(x) = \begin{cases} 3+2x, -\frac{3}{2} < x \leq 0 \\ 3-2x, \ 0 < x < \frac{3}{2} \end{cases}$

24. Prove that between any two real roots of e^x sin x =1 , there exists at least one real root of e^xcos x +1 =0 .

25. Expand sin x , $x \in \mathbb{R}$ in powers of x by Tailor's series expansion.

26. Find the minimum value (if exists) of the function defined by f(x) $=x^3, (x > 0)$.

27. Show that the greatest value of $x^m y^n$, (x>0 ,y>0) and x+y= k (k=constant) is $\frac{m^n}{}$

29. Prove that between any two real roots of $e^x \sin x$ =1 there exist at least one real root of $e^x \cos x$ +1 =0 .

30. A function f is thrice differentiable on *[a, b]* and *f(a) =f(b) =0* and also f'(a)=f'(b)=0. Prove that the second derivative of f vanishes at c, where a$< c < b$.

31. Define discrete and pseudo metric space.

32. On the Real line $\mathbb{R}$, show that a singleton set is not an open set.

33. Let X be the set of all sequences of all real numbers containing only a finite number of non- zero element. Let d: $X \times X \to$X be defined by d $(\{x_n\},\{y_n\})$ $=\{\sum_{r=1}^{\infty}(x_r - y_r)^2\}^{\frac{1}{2}}$.

34. Give an example to show that the continuous image of an open bounded interval may not be an open bounded interval.

35. in the mean value theorem (a) in the mean value theorem f (x+h) =f (x)+h$f^{/}$ (x+θh), 0<θ<1 ,prove that $lim_{h\to 0}\ \theta$=$\frac{1}{2}$ if f (x) =sin x

36. Show that the any discrete metric space is complete metric space.

37. Show that in any metric space a finite set has no limit point

38. Show by example that in any metric space .The cantor intersection theorem may not hold

good if any of the following condition is not satisfied

i) $\{F_n\}$ is a sequence of closed sets

ii) δ (F_n) →0 as n→ ∞ where δ(A) denotes the diameter of the set A

39. We know in a metric space (X,d) , "the union of a finite number of closed set is closed in this result if we drop the finiteness , then is the result hold good ? justify.

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